# Energy Efficiency of Distributed Massive MIMO Systems

Chunlong He, Jiajia Yin, Yejun He, Min Huang, and Bo Zhao

***Abstract:*** **In this paper, we investigate** *energy efficiency* **(EE) of the traditional co-located and the distributed massive** *multiple-input multiple-output* **(MIMO) systems. First, we derive an approximate EE expression for both the idealistic and the realistic power consumption models. Then an optimal** *energy-efficient remote access unit* **(RAU) selection algorithm based on the distance between the** *mobile stations* **(MSs) and the RAUs are developed to maximize the EE for the downlink distributed massive MIMO systems under the realistic power consumption model. Numerical results show that the EE of the distributed massive MIMO systems is larger than the co-located massive MIMO systems under both the idealistic and realistic power consumption models, and the optimal EE can be obtained by the developed** *energy-efficient* **RAU selection algorithm.**

***Index Terms:*** **Co-located massive multiple-input multiple-output (MIMO), distributed massive MIMO, energy efficiency.**

## I. INTRODUCTION

DISTRIBUTED massive *multiple-input multiple-output* (MI MO) systems can take advantages of both the *distributed antenna systems* (DAS) and the point-to-point MIMO, so its performance can be improved by exploiting the spatial macro and micro diversities. Compared with the traditional co-located massive MIMO [1]–[3], the distributed massive MIMO systems can reduce the access distance between the *remote access units* (RAUs) and the *mobile stations* (MSs), which means lower propagation losses and higher spatial reuse. Thus the distributed massive MIMO has the advantages of increasing ergodic sum rate [4]–[12], extending coverage [13]–[15], and improving *energy efficiency* (EE) [16]–[20].

Most of the recent research on distributed massive MIMO systems has focused on ergodic sum rate. A greedy user scheduling algorithm has been proposed in [21] to improve the ergodic sum rate in the downlink distributed massive MIMO systems. By jointly optimizing the input covariances of all MSs, an iterative algorithm has been developed in [22] to maximize the ergodic sum rate in the multi-cell downlink distributed massive MIMO systems. Based on random matrix theory [23], [24], a closed-form capacity has been derived in [25] for the uplink distributed massive MIMO systems. Due to the dramatic growth in high data rate transmission and multimedia services driven by using smart iPhone and Android devices, tablets, ebook readers, and other wireless devices, a large amount of energy has been consumed. It has been forecasted in [26] that the global mobile data traffic will grow further by over 100 times in the following ten years. So EE is becoming more and more important in the 5G wireless communication systems. The EE performance has been discussed in [27] and [28] for the traditional co-located massive MIMO systems. However, the distributed massive MIMO systems suffer from different degrees of path losses caused by different access distances between the MS and the RAUs. As a result, the existing EE results for the traditional co-located massive MIMO systems cannot be directly applied to the distributed massive MIMO systems.

In our previous research work in [16], a Pareto optimal solution of EE by exploiting the multi-criteria optimization method has been developed for DAS. In [19], we have also demonstrated that DAS systems are more energy-efficient than *co-located antenna systems* (CAS) systems. However both of our previous work only considered small scale antennas for DAS. In this paper, we investigate the EE of the traditional co-located and the distributed massive MIMO systems. The channel model considered in this paper includes small scale and large scale fading due to the physically separated RAUs. We derive an approximate EE expression for both the idealistic and the realistic power consumption models, which will be introduced in Section II. B. Because every employed antenna corresponding to a separate radio frequency chain [29], so the total transmit power consumption of the downlink distributed massive MIMO systems scales linearly with the number of the transmit antenna. Thus when the number of the RAU goes to infinite, then the total realistic power consumption also goes to infinity and the hardware complexity is also very high, which is impractical to use all the RAUs to achieve high capacity due to the extensive power consumption and expensive radio frequency chains. So, based on the distance between the MS and the RAUs, an optimal *energy-efficient* RAU selection algorithm is developed to maximize the EE for the downlink distributed massive MIMO systems under the realistic power consumption model. The main contributions of this paper can be summarized as follows,

- We derive an approximate EE expression for both the idealistic and the realistic power consumption models.
- The EE of the downlink distributed massive MIMO systems first increases and then decreases with the increase of the maximum transmit power or the number of RAU when considering the realistic power consumption model.

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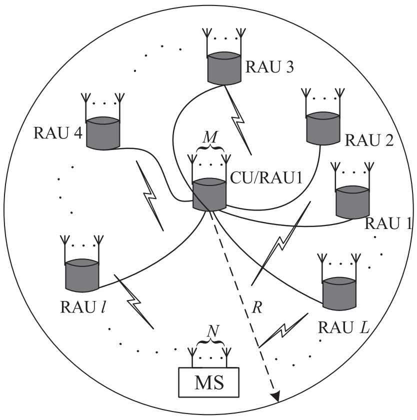


Fig. 1. Circular layout of a single cell distributed massive MIMO configuration.

- For the idealistic power consumption model, the EE decreases with the increase of the maximum transmit power. However, the EE increases with the increase of the number of the RAU.
- Based on the distance between the MS and the RAUs, an optimal *energy-efficient* RAU selection algorithm is developed to maximize the EE under the realistic power consumption model.

The rest of the paper is organized as follows. The system models are presented in Section II. Then the EE of the distributed massive MIMO systems and the co-located massive MIMO systems are discussed in Sections III and IV, respectively. Numerical results are presented to evaluate the EE performance in Section V. Section VI concludes the paper.

*Notations*: In this paper, $(.)^T$, $(.)^H$ denote transpose and Hermitian transpose, respectively. $\mathbf{I_k}$ denotes identity matrix of size $k \times k$. det($\mathbf{A}$) denotes the determinant of $\mathbf{A}$. The operator $E(\cdot)$ denotes expectation.

## II. SYSTEM MODELS

After briefly discussing the distributed massive MIMO systems and the realistic and the idealistic power consumption models, we introduce the EE of the downlink distributed and the co-located massive MIMO systems.

### A. Distributed Massive MIMO Systems Model

Fig. 1 shows the distributed massive MIMO systems in a single cell with $L$ RAUs. For simplicity of analysis, we only consider a single user scenario in this paper. We assume that the MS and the RAUs are equipped with $N$ antennas and $M$ antennas, respectively. We denote this system configuration as $(N, M, L)$ [30]. When $L = 1$, it becomes a traditional co-located massive MIMO systems. The *central unit* (CU) can be regarded as a special RAU and is denoted by RAU 1. The RAUs are low-power and low-cost BSs, and only equipped with up/down converters and low-noise amplifiers, which are all physically connected to the CU through the optical fibers. We assume that the channel state information is unkonwn at the transmitter side but completely known at the receiver side. The overall received signal of the downlink distributed massive MIMO is given by

$$Y = \mathbf{H}(\mathbf{d})\mathbf{X} + \mathbf{n} \tag{1}$$

where $\mathbf{Y}$ and $\mathbf{X}$ are the $N \times 1$ dimensional received signal vector and the $ML \times 1$ dimensional transmitted signal vector, respectively. $\mathbf{n}$ is the $N \times 1$ dimensional complex *additive white Gaussian noise* (AWGN) vector with covariance matrix $E(\mathbf{nn}^H) = \sigma_z^2 \mathbf{I_N}$, $\sigma_z^2$ denotes the power of the AWGN. $d = [d_1, d_2, \cdots, d_L]^T$ is the distance vector from the $l$-th RAU to the MS. $\mathbf{H}(\mathbf{d}) = [\mathbf{H_1}(\mathbf{d_1}), \cdots, \mathbf{H_l}(\mathbf{d_l}), \cdots, \mathbf{H_L}(\mathbf{d_L})]$ is the $N \times ML$ dimensional channel frequency response, which includes small scale and large scale fading channel, and can be modeled as [14]

$$\mathbf{H_l}(\mathbf{d_l}) = h_{sh,l}\mathbf{H_{w,l}}, \qquad \mathbf{1} \le \mathbf{l} \le L \tag{2}$$

where $\mathbf{H_{w,l}}$ is the small scale fading channel and is an independent and identically distributed complex Gaussian random variables with zero mean and unit variance [16], and $h_{sh,l}$ is the large scale fading and is independent of $\mathbf{H_{w,l}}$. The large scale fading can be written as [14]

$$h_{sh,l} = \sqrt{\frac{cs_l}{d_l^\alpha}} \tag{3}$$

where $c$ is the median of the mean path gain at a reference distance $d_l$ = 1 km, $d_l$ is the distance between the RAU $l$ and the MS, $\alpha$ is the path loss exponent and is typically between 3 and 5, and $s_l$ is a log-normal shadow fading variable, i.e., $10 \log_{10} s_l$ is a zero-mean Gaussian random variable with standard deviation $\sigma_{sh}$ [14].

For simplicity of analysis, we assume that the MS is uniformly distributed in the cell and the cell shape is approximated by a circle of radius $R$. $(D_l, \theta_l)$ denotes the RAU's polar coordinates relative to the center of the cell, the RAU's location are the same as [31], that is, the CU/RAU 1 polar coordinate is $(0, 0)$, and the other RAU's polar coordinates are $\left((3-\sqrt{3})R/2, 2\pi(l-1)/(L-1)\right), l = 1, 2, \cdots, L-1$. $(\rho, \theta)$ denotes the MS's polar coordinate. So the distance $d_l$ between the $l$-th RAU and the MS can be calculated as [14]

$$d_l = \sqrt{\rho^2 + D_l^2 - 2\rho D_l \cos(\theta - \theta_l)}. \tag{4}$$

The probability density functions of the MS's polar coordinate $(\rho, \theta)$ are given by [14]

$$p(\rho) = \frac{2\rho}{R^2}, \qquad 0 \le \rho \le R, \tag{5}$$

$$p(\theta) = \frac{1}{2\pi}, \qquad 0 \le \theta \le 2\pi. \tag{6}$$

### B. Total Power Consumption Model

As we have discussed in [31], the realistic power consumption in the distributed massive MIMO systems consists of four parts and can be expressed as,

$$P_{\text{Realistic}}^D = \frac{P_t}{\tau} + MLP_{\text{dyn}} + P_{\text{sta}} + P_o \tag{7}$$

where $P_t$ is the overall transmit power and $\tau$ denotes the radio frequency power amplifier efficiency, $P_{\text{dyn}}$, $P_{\text{sta}}$, and $P_o$ are the dynamic power consumption, the static power consumption, and the dissipated power consumption by the optical fiber transmission [32], respectively.

When the circuit power is ignored, the idealistic power consumption model can be written as,

$$P^D_{\text{Idealistic}} = \frac{P_t}{\tau}. \tag{8}$$

For the co-located massive MIMO systems, the realistic power consumption can be expressed as,

$$P^C_{\text{Realistic}} = \frac{P_t}{\tau} + MP_{\text{dyn}} + P_{\text{sta}}. \tag{9}$$

The difference between the distributed and the co-located massive MIMO systems is the optical fiber power consumption. The idealistic power consumption model of the co-located massive MIMO systems is the same as that of the distributed massive MIMO systems in (8).

### C. EE Model

As in [31], [33]–[36], the EE of the co-located and the distributed massive MIMO systems is defined as the ratio of the capacity and the total power consumption,

$$\eta_{EE}(C) = \frac{C}{P_{\text{Total}}} \tag{10}$$

where $C$ is the capacity of the co-located or the distributed massive MIMO systems. $P_{\text{Total}}$ is equal to (7) for the distributed massive MIMO systems or (9) for the co-located massive MIMO systems when considering the realistic power consumption model, otherwise $P_{\text{Total}}$ is equal to (8).

## III. EE OF THE DISTRIBUTED MASSIVE MIMO SYSTEMS

In this section, we will first derive a closed-form approximate EE expression and the corresponding properties for the distributed massive MIMO systems. Then an optimal *energy-efficient* RAU selection algorithm based on the distance between the MS and the RAUs will be developed to maximize the EE under the realistic power consumption model.

### A. EE of the Distributed Massive MIMO

The mutual information of the downlink distributed massive MIMO systems can be written as [14]

$$I = \log_2 \det \left[ \mathbf{I_N} + \frac{P_\mathbf{t}}{ML\sigma^2_\mathbf{z}} \mathbf{H}(\mathbf{d})\mathbf{H}^\mathbf{H}(\mathbf{d}) \right]. \tag{11}$$

For the downlink distributed massive MIMO systems, we can conclude that $N \leq ML$ and $ML$ is very large, and the distances from the MS to the RAUs are known, we have the following results, which have been partly proved in [14] and for the better understand we rewritten it in Appendix A.

***Theorem** 1:* For the ($N$, $M$, $L$) downlink distributed massive MIMO systems, the approximate capacity expression can be expressed as following [14],

$$\begin{aligned}\overline{C}_{\text{D\_M\_MIMO}} \approx{} & N\left(\log_2 \frac{cP_t}{R^\alpha \sigma_z^2} + \log_2 \exp\left(-\frac{(4-\sqrt{3})\alpha}{4}\right)\right) \\ & + N\left(\log_2 L + \sum_{i=0}^{M-1} \log_2 \Gamma(M-i+\frac{1}{N})\right) \\ & + N\left(\frac{\alpha + \lambda^2\sigma_{sh}^2}{2\ln 2} - \sum_{i=0}^{M-1} \log_2 \Gamma(M-i)\right),\end{aligned} \tag{12}$$

where $\Gamma(\cdot)$ is the Gamma function.

So according to (10), the EE of the distributed massive MIMO systems can be expressed as

$$\eta_{\text{D\_M\_MIMO\_EE}} = \frac{\overline{C}_{\text{D\_M\_MIMO}}}{P^D_{\text{Total}}}. \tag{13}$$

### B. Properties of the EE

From (13), we can obtain the following EE property for the downlink distributed massive MIMO systems which can be expressed as in Theorem 2, and is proved in Appendix B.

***Theorem** 2:* For the realistic power consumption model, the EE of the downlink distributed massive MIMO systems first increases and then decreases with the increase of the maximum transmit power. However, for the idealistic power consumption model, the EE decreases with the increase of the maximum transmit power.

From Theorem 2, for the realistic power consumption model, the EE first increases and then decreases with the increase of the maximum transmit power. So there is a tradeoff between the EE and the maximum transmit power or data rate. The reason is, when the maximum transmit power is small, the growth rate of the capacity is greater than the growth rate of the maximum transmit power, thus the increase of the capacity is enough to compensate for the increase of the realistic power consumption. However when the maximum transmit power is large, the growth rate of the maximum transmit power is greater than the growth rate of the capacity, thus the increase of the capacity is not enough to compensate for the increase of the realistic power consumption. The above tradeoff phenomenon has been also found in SISO systems [36], [37]. But for the idealistic power consumption model, the EE is a decreasing function of the maximum transmit power. In that case, the growth rate of the maximum transmit power is always greater than the growth rate of the capacity, thus the increase of the capacity is not enough to compensate for the increase of the idealistic power consumption, which has been also found in *single-input single-output* (SISO) systems in [37].

Theorem 2 addresses EE with regard to the maximum transmit power. In the following Theorem 3, we will discuss the EE with regard to the number of the RAUs when considering the realistic and idealistic power consumption model, respectively, which is proved in Appendix C.

***Theorem** 3:* For the realistic power consumption model, the EE of the downlink distributed massive MIMO systems first increases and then decreases with the increase of the number of the RAU. However, for the idealistic power consumption model, the EE increases with the increase of the number of the RAU.

From Theorem 3, for the realistic power consumption model, the EE first increases and then decreases with the increase of the number of the RAU. So there is a tradeoff between the EE and the number of the RAU. The reason is, when the number of the using RAU is large than the optimal number of the RAU that maximize the EE, the increase of the capacity is not enough to compensate for the increase of the realistic power consumption. However for the idealistic power consumption model, the EE is an increasing function of the number of the RAU. The reason is an increase number of the RAU can improve the capacity significantly but has little contribution to the idealistic power consumption.

### *C. Energy-Efficient RAU Selection Algorithm*

From Theorem 3, for the realistic power consumption model, there is a tradeoff between the EE and the number of the RAU. The realistic power consumption scales linearly with the number of the RAU. Thus when the number of the RAU goes to infinite, then the total realistic power consumption also goes to infinity. So it is impractical to use all the RAUs to achieve high capacity or the EE due to the extensive power consumption and expensive radio frequency chains. According to the channel model in (2), channel gain is related to the distance between the transmit and receiver. So an optimal *energy-efficient* RAU selection algorithm based on the distance between the MS and the RAUs is developed in Table I.

**Algorithm 1.** Energy-efficient RAU selection algorithm

Initialization $\mathcal{K} = [\,]$, $\eta_{EE}(\mathcal{K})$=0 and $E_{Ch}$ is the number of the available radio frequency chains;
while $\|\mathcal{K}\|$<$E_{Ch}$
  find $(l) = \arg\min\{d_1, d_2, \cdots, d_L\}$;
  Let $\mathcal{K}'$=$\mathcal{K}$+$\{l\}$, and $d_l = 3R$;
  Calculate $\eta_{EE}(\mathcal{K}')$ according to (13);
  if $\eta_{EE}(\mathcal{K}') \leq \eta_{\mathcal{EE}}(\mathcal{K})$
    break;
  else
    Let $\mathcal{K}' = \mathcal{K}$, and $\eta_{EE}(\mathcal{K}) = \eta_{\mathcal{EE}}(\mathcal{K}')$.
  end if
end while.
Return $\mathcal{K}$ and $\eta_{EE}(\mathcal{K})$

In Algorithm 1, $\mathcal{K}$ is the current optimal set of the RAU after each loop. First we initialize $\mathcal{K}$ as empty. Then we update $\mathcal{K}$ by trying to add one or more antennas through the while loop. Considering the constraint of the radio frequency chains, the algorithm terminates if $\|\mathcal{K}\|$ is larger than $E_{Ch}$, the number of the available radio frequency chains. In each loop, we first trying to find the smallest distance between the MS and the RAUs, which corresponding to the best channel condition. Then we add this selected RAU to form a new set $\mathcal{K}'$ and let the distance between the MS and the selected RAUs be $3R$, which is to ensure this selected RAU will not be chosen again in the next loop. If $\eta_{EE}(\mathcal{K}')$ is smaller or equals to $\eta_{EE}(\mathcal{K})$, the algorithm terminates, otherwise let $\mathcal{K}' = \mathcal{K}$ and $\eta_{EE}(\mathcal{K}) = \eta_{\mathcal{EE}}(\mathcal{K}')$. The algorithm return the optimal set of the RAU $\mathcal{K}$ and the optimal EE, $\eta_{EE}(\mathcal{K})$, when the algorithm terminates. The search size of the algorithm in Table I is $E_{Ch} - 1$, so it is easy to implement in the practical systems.

## IV. EE OF A CO-LOCATED MASSIVE MIMO MODEL

The mutual information of the downlink collocated massive MIMO systems can be written as [14]

$$I = \log_2 \det \left[ \mathbf{I_N} + \frac{\mathbf{c}P_\mathbf{t}}{M\sigma_\mathbf{z}^2} \frac{\mathbf{s}}{d^\alpha} \mathbf{H_w H_w^H} \right] \tag{14}$$

where $\mathbf{H_w}$ is the $N \times M$ dimensional small scale fading channel between the MS and the CU. $d$ is the distance between the MS and the CU. From [14], the average capacity can be written as

$$\begin{aligned} \overline{C}_{C\_M\_MIMO} \approx N \log_2 \frac{cP_t}{\sigma_z^2} - \alpha N \log_2 R + \frac{\alpha N}{2 \ln 2} \\ + \frac{1}{\ln 2} \sum_{i=0}^{N-1} \psi(M-i) \end{aligned} \tag{15}$$

where $\psi(\cdot)$ is the Euler's function. Note that we cannot simply set $L = 1$ in (12) to obtain the average capacity expression for the co-located case. The reason is (12) and (15) are derived from different methods. From Fig. 2, shown at the next page, we can conclude that, for the co-located case, the results of (15) is almost the same as the theoretical expression, but for the distributed case, the results of (12) is not very close to the theoretical expression. So for the co-located case, if we simply set $L = 1$ in (12) to obtain the results for the co-located case, then the approximate capacity simulation results and theoretical capacity simulation results will have some error.

According to (10), the EE of the co-located massive MIMO systems can be expressed as

$$\eta_{C\_M\_\mathrm{MIMO\_EE}} = \frac{\overline{C}_{C\_\mathrm{M\_MIMO}}}{P_{\mathrm{Total}}^C}. \tag{16}$$

$P_{\mathrm{Total}}^C$ is equal to (9) when considering the realistic power consumption model, otherwise $P_{\mathrm{Total}}^C$ is equal to (8). The EE of the co-located massive MIMO systems has the similar properties as the distributed massive MIMO systems, and their proofs are omitted here.

## V. NUMERICAL RESULTS

In this section, numerical results are presented to evaluate the performance of the downlink distributed massive MIMO systems under the different power consumption models. The related circuit and system parameters used in our simulation are listed in Table II.

Table 1. Simulation parameters.

| Parameters | Value |
|---|---|
| The noise power $\sigma_z^2$ | $-104$ dBm |
| Cell radius $R$ | 1 km |
| The dynamic power consumption $P_{\rm dyn}$ | 30 dBm [38], [39] |
| The static power consumption $P_{\rm sta}$ | 40 dBm [38], [39] |
| The maximum transmit power $P_t$ | 46 dBm |
| Path loss exponent $\alpha$ | 4 |
| Drain efficiency $\tau$ | 38% [29] |
| The optical Fiber dissipated power $P_o$ | 0.1484 dB/km [32] |
| Shadow fading $\sigma_{sh}$ | 8 dB |

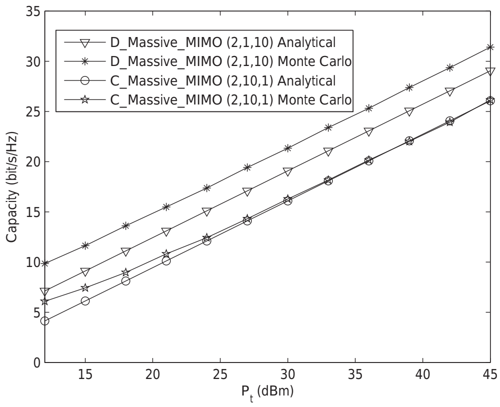


Fig. 2. Capacity versus the maximum transmit power.

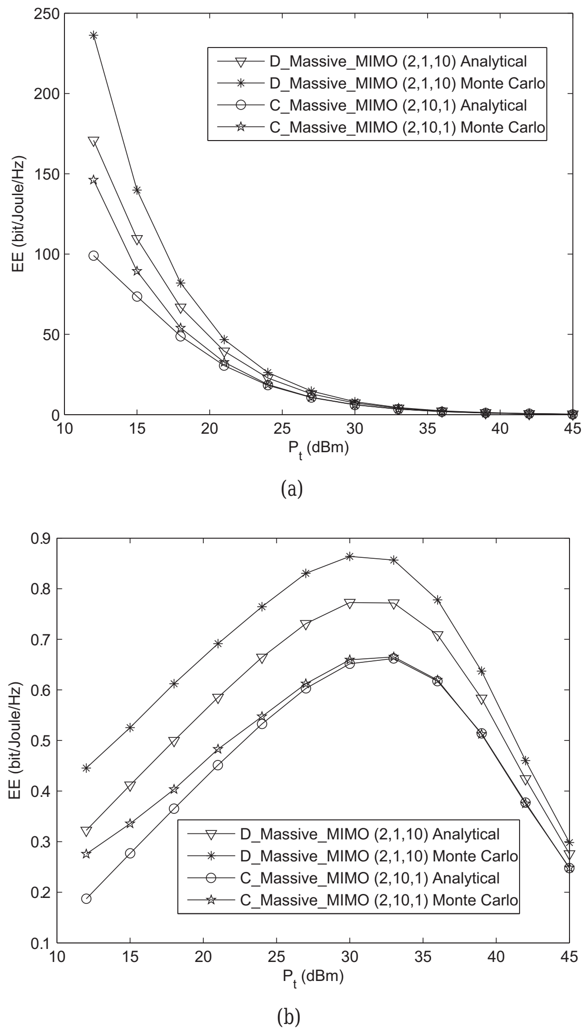


Fig. 3. Energy efficiency versus the maximum transmit power with different power consumption model: (a) Idealistic power consumption model and (b) realistic power consumption model.

Figs. 2 and 3 compare the capacity and EE for the downlink distributed and co-located massive MIMO systems, respectively, under the different power consumption models. From Fig. 2, compared with empirical results, the closed-form approximate capacity in (15) for the co-located massive MIMO systems is very accurate when the maximum transmit power is large. And the closed-form approximate capacity in (12) for the distributed massive MIMO systems is also nearly accurate over all the maximum transmit power levels. From Fig. 3(a), we can see that the EE decreases with the increase of the maximum transmit power under the idealistic power consumption model. And from Fig. 3(b), the EE first increases and then decreases with the increase of the maximum transmit power under the realistic power consumption model, which agree with Theorem 2. We conclude that compared with the co-located massive MIMO systems, the distributed massive MIMO systems can significantly improve the EE. The reason is that the average access distance between the MS and the RAU is decreased. So the transmit power is also decreased and the EE of the distributed massive MIMO systems is increased. As we can see from Fig. 3, the EE of the distributed massive MIMO systems is approximately 23.1% higher than the co-located MIMO system when the maximum transmit power is 30 dBm.

Fig. 4 compares the EE versus the number of the RAU for the downlink massive MIMO systems under the realistic and idealistic power consumption models. From Fig. 4, we can see that the EE increases with the increase of the number of the RAU under the idealistic power consumption model, and the EE first increases and then decreases with the increase of the number of the RAU under the realistic power consumption model, which agree with Theorem 3.

Fig. 5 compares the capacity and the EE under the proposed *energy-efficient* RAU selection algorithm. The traditional method denotes all the RAUs are activated in the systems. In this case, the capacity of the traditional method is better than the proposed *energy-efficient* RAU selection algorithm. As we can see from Fig. 5(a), the capacity of the traditional method is approximately 8.2% higher than the proposed *energy-efficient* RAU selection algorithm when the maximum transmit power is 30 dBm. However, the performance of the proposed *energy-efficient* RAU selection algorithm is better than the traditional method in terms of the EE. From Fig. 5(b), the EE of the

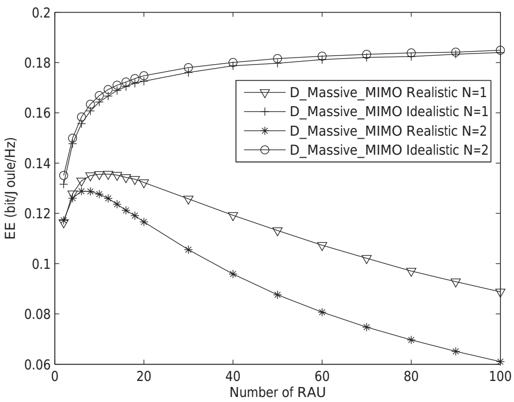


Fig. 4. EE versus number of RAU with $M = 1$ and $P_t = 46$ dBm.

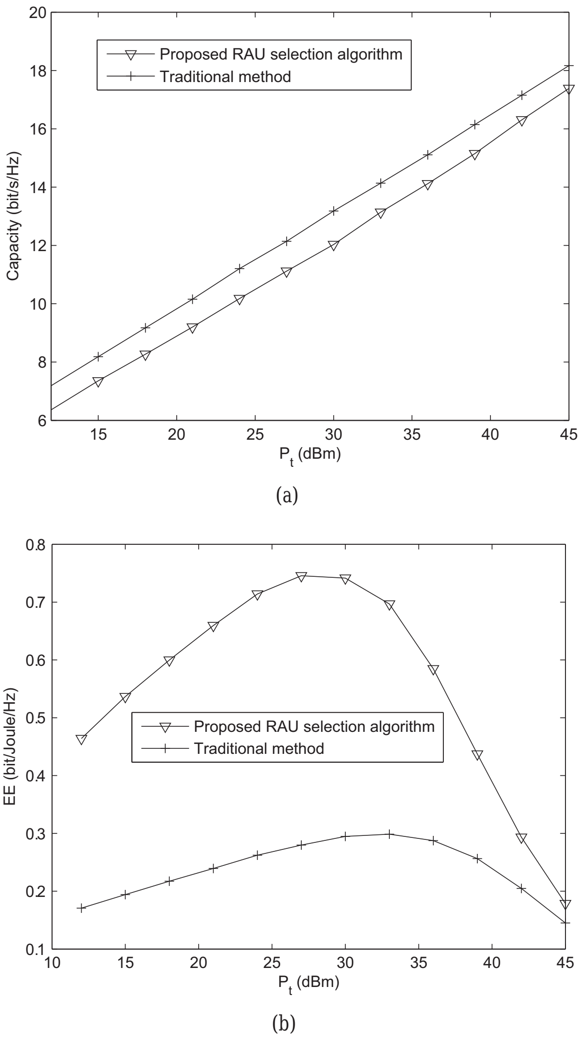


Fig. 5. Capacity & energy efficiency versus the maximum transmit power for the RAU selection algorithm with $N = 1$, $M = 1$, $L = 30$, and $E_{ch} = 20$.

proposed *energy-efficient* RAU selection algorithm is approximately 146.1% higher than the traditional method.

## VI. CONCLUSIONS

In this paper, we have investigated EE for the traditional co-located and the distributed massive MIMO systems. An approximate EE expression was derived for both the idealistic and the realistic power consumption models. Based on the distance between the MS and the RAUs, an optimal *energy-efficient* RAU selection algorithm was developed to maximize the EE for the realistic power consumption model. Simulation results have demonstrated the effectiveness of the proposed algorithms, and the EE of the distributed massive MIMO systems is larger than the co-located massive MIMO systems under both the idealistic and realistic power consumption models. For future research, energy-efficient resource allocation algorithm when multiple users and multiple cells are deployed need to be addressed for both the co-located and the distributed massive MIMO systems.

## APPENDIX A: Proof of Theorem 1

When $N \leq ML$ and $ML$ is very large, and the distances from the MS to the RAUs are known, firstly we assume that the MS's position is fixed, by exploiting the Minkowski inequality [40], [14], we have

$$
\begin{aligned}
C(\rho,\theta) \geq\; & N E_{s,\mathbf{H_w}} * \\
& \left\{ \log_2 \left[ 1 + \frac{cP_t}{\sigma_z^2} \sum_{l=1}^{L} \frac{s_l}{d_l^\alpha} \det(\mathbf{H_{w,l}} \mathbf{H_{w,l}^H})^{\frac{1}{N}} \right] \right\} \\
& \approx N \log_2 \sum_{l=1}^{L} \frac{1}{d_l^\alpha} E_s(s_l) E_{\mathbf{H}} \left[ \det(\mathbf{H_{w,l}} \mathbf{H_{w,l}^H})^{\frac{1}{N}} \right] \\
& + N \log_2 \frac{cP_t}{\sigma_n^2}.
\end{aligned} \tag{17}
$$

Applying Theorem 2.11 in [23], we have

$$
E_{\mathbf{H_w}} \left[ \det(\mathbf{H_{w,l}^H} \mathbf{H_{w,l}})^{\frac{1}{N}} \right] = \frac{\prod_{i=0}^{M-1} \Gamma(M - i + \frac{1}{N})}{\prod_{i=0}^{M-1} \Gamma(M - i)} \tag{18}
$$

where $\Gamma(\cdot)$ is the Gamma function.
Then for the fixed MS position, we can get the average capacity of the downlink distributed massive MIMO systems as [14]

$$
\begin{aligned}
C(\rho,\theta) = N & \left( \log_2 \frac{cP_t}{\sigma_n^2} + \log_2 \sum_{l=1}^{L} \frac{1}{d_n^\alpha} + \frac{\lambda^2 \sigma_{sh}^2}{2 \ln 2} \right) \\
& + N \left( \sum_{i=0}^{M-1} \log_2 \Gamma(M - i + \frac{1}{N}) \right) \\
& - N \left( \sum_{i=0}^{M-1} \log_2 \Gamma(M - i) \right).
\end{aligned} \tag{19}
$$

When the MS's position is random, according to the reference [14], (5), and (6), we have

$$
\begin{aligned}
E_{\rho,\theta} \left\{ \log_2 \sum_{l=1}^{L} d_l^{-\alpha} \right\} \simeq \log_2 \sum_{l=1}^{L} \exp \left( -\frac{\alpha D_l^2}{2R^2} \right) \\
+ \frac{\alpha}{2 \ln 2} - \alpha \log_2(R).
\end{aligned} \tag{20}
$$

Substituting $D_l = (3-\sqrt{3})R/2$ and (20) into (19), we can obtain (12) immediately, this completes the proof of Theorem 1.

## APPENDIX B: Proof of Theorem 2

For the realistic power consumption model, from (13), the EE of the downlink distributed massive MIMO systems can be expressed as

$$\eta_{\text{D\_M\_MIMO\_EE}} = \frac{N\left(\log_2 \frac{cP_t}{R^\alpha \sigma_z^2} + \log_2 L + T1\right)}{\frac{P_t}{\tau} + MLP_{\text{dyn}} + P_{\text{sta}} + P_o} \tag{21}$$

where

$$T1 = \log_2 \exp\left(-\frac{(4-\sqrt{3})\alpha}{4}\right) + \frac{\alpha + \lambda^2 \sigma_{sh}^2}{2\ln 2} + \sum_{i=0}^{M-1} \log_2 \Gamma(M-i+\frac{1}{N}) - \sum_{i=0}^{M-1} \log_2 \Gamma(M-i).$$

After differentiating with respect to the maximum transmit power $P_t$, we have

$$\frac{\partial \eta_{\text{D\_M\_MIMO\_EE}}}{\partial P_t} = \frac{T2 - \frac{N}{\tau}\left(\log_2 \frac{cP_t}{R^\alpha \sigma_z^2} + \log_2 L + T1\right)}{\left(\frac{P_t}{\tau} + MLP_{\text{dyn}} + P_{\text{sta}} + P_o\right)^2} \tag{22}$$

where

$$T2 = \frac{N}{P_t \ln 2}\left(\frac{P_t}{\tau} + MLP_{\text{dyn}} + P_{\text{sta}} + P_o\right).$$

The denominator of (22) is a positive value, the sign of (22) is determined by the numerator. Let the numerator be

$$f(P_t) = T2 - \frac{N}{\tau}\left(\log_2 \frac{cP_t}{R^\alpha \sigma_z^2} + \log_2 L + T1\right). \tag{23}$$

Taking the first derivative with respect to $P_t$, we observe

$$\frac{\partial f(P_t)}{\partial P_t} = -T2 - \frac{N}{\tau P_t \ln 2} < 0. \tag{24}$$

So $f(P_t)$ is a decreasing function with respect to the maximum transmit power $P_t$, thus we have $f(\infty) \leq f(P_t) \leq f(0)$. From (23), we can easily obtain $\lim\limits_{P_t \to 0} f(P_t) > 0$ and $\lim\limits_{P_t \to \infty} f(P_t) < 0$.

Then (23) is positive when the maximum transmit power $P_t$ is small and is negative when $P_t$ is large. Thus we can get the conclusion that the EE of the downlink distributed massive MIMO systems first increases and then decreases with the increase of the maximum transmit power $P_t$. Exploiting the similar method to above, we can get the conclusion that the EE decreases with the increase of the maximum transmit power $P_t$ when considering the idealistic power consumption model. This completes the proof of Theorem 2.

## APPENDIX C: Proof of Theorem 3

For the realistic power consumption model, the EE of the downlink distributed massive MIMO systems is the same as (21). By differentiating with respect to the number of the RAU $L$, we have

$$\frac{\partial \eta_{\text{D\_M\_MIMO\_EE}}}{\partial L} = \frac{\frac{P_t T2}{L} - N\left(\log_2 \frac{cP_t}{R^\alpha \sigma_z^2} + \log_2 L + T\right) MP_{\text{dyn}}}{\left(\frac{P_t}{\tau} + MLP_{\text{dyn}} + P_{\text{sta}} + P_o\right)^2}. \tag{25}$$

The denominator of (25) is a positive value, the sign of the above (25) is determined by the numerator. Let the numerator as

$$g(L) = \frac{P_t T2}{L} - N\left(\log_2 \frac{cP_t}{R^\alpha \sigma_z^2} + \log_2 L + T\right) MP_{\text{dyn}}. \tag{26}$$

Taking the first derivative with respect to $L$, we observe

$$\frac{\partial g(L)}{\partial L} = -\frac{N}{L^2 \ln 2}\left(\frac{P_t}{\tau} + P_{\text{sta}} + P_o\right) - \frac{NMP_{\text{dyn}}}{L \ln 2} < 0. \tag{27}$$

So $g(L)$ is a decreasing function with respect to the number of the RAU $L$, thus we have $g(\infty) \leq g(L) \leq g(0)$. From (26), we can easily obtain $\lim\limits_{L \to 0} g(L) > 0$ and $\lim\limits_{P_t \to \infty} g(L) < 0$.

Then (25) is positive when the number of the RAU $L$ is small and is negative when $L$ is large. Thus we can get the conclusion that the EE of the downlink distributed massive MIMO systems first increases and then decreases with the increase of the number of the RAU $L$. Exploiting the similar method as above, we can get the conclusion that the EE decreases with the increase of the number of the RAU $L$ when considering the idealistic power consumption model. This completes the proof of Theorem 3.

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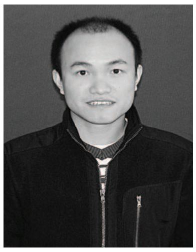

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